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Describing a_1 and b_1 decays

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Abstract

Two-body pion-radiating and weak decays of light axial-vector mesons and the ρ are studied as a phenomenological application of the QCD Dyson-Schwinger equations. Models based on the rainbow-ladder truncation are capable of providing a good description and, in particular, yield the correct sign and magnitude of the $a_1 \to \rho\pi$ and $b_1 \to \omega\pi$ D/S ratios, with no additional mechanism necessary.

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The light meson spectrum contains [1] four little-studied axial-vector mesons composed of u- and d-quarks. They appear as isospin $I=0,1$ partners (in the manner of the ω and ρ): $h_1(1170)$, $b_1(1235)$; and $f_1(1285)$, $a_1(1260)$, and differ in their charge-parity: $J^{PC}=1^{+-}$ for h_1, b_1; and $J^{PC}=1^{++}$ for f_1, a_1. In the $q\bar{q}$ constituent quark model the b_1 is represented as a constituent-quark and -antiquark with total spin $S=0$ and angular momentum $L=1$, while in the a_1 the quark and antiquark have $S=1$ and $L=1$. It is therefore apparent that in this model the b_1 is an orbital excitation of the π, and the a_1 an orbital excitation and axial-vector partner of the ρ. In QCD the J^{PC} characteristics of a quark-antiquark bound state are manifest in the structure of its Bethe-Salpeter amplitude [2]. This amplitude is a valuable intuitive guide and, in cases where a $q\bar{q}$ constituent quark model analogue exists, it incorporates and extends the information present in that analogue's quantum mechanical wave function. We describe mesons via their Bethe-Salpeter amplitudes.

Three of the axial-vector mesons decay predominantly into two-body final states containing a vector meson and a pion:[1] $h_1\to\rho\pi$; $b_1\to\omega\pi$; $a_1\to\rho\pi$, and with a $J=1$ meson in both the initial and final state they proceed via two partial waves (S, D). Such decays therefore probe aspects of hadron structure inaccessible in simpler processes involving only spinless mesons in the final state, such as $\rho\to\pi\pi$; e.g., in constituent-quark-like models the D/S amplitude ratio is very sensitive [3] to the nature of the phenomenological long-range confining interaction. The additional insight and model constraints that such processes can provide is particularly important now as a systematic search and classification of "exotic" states[2] in the light meson sector becomes feasible experimentally. Therefore, as an illustration of the application [7] of the QCD Dyson-Schwinger equations (DSEs) [8], and an exploration and further elucidation of the domain of applicability of commonly used truncations, we report a simultaneous study of these axial-vector meson decays, the $\rho\to\pi\pi$ decay, and the leptonic decay constants. The breadth of the study ensures a global view.

In the isospin symmetric limit the homogeneous Bethe-Salpeter equation (BSE) for a quark-antiquark bound state is

$$\Gamma^{tu}(k;P)=\int_q^\Lambda \chi^{sr}(q;P)\,K^{rs}_{tu}(q,k;P), \tag{1}$$

where k is the relative and P the total momentum of the constituents, $\chi(q;P):=S(q_+)\Gamma(q;P)S(q_-)$, $r,\ldots,u$ represent colour, Dirac and isospin indices, $q_\pm=q\pm P/2$, and $\int_q^\Lambda:=\int^\Lambda d^4q/(2\pi)^4$ represents mnemonically a translationally invariant regularisation of the

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[1] $f_1\to\rho\pi$ is forbidden by G-parity conservation and $f_1\to\omega\pi$ does not conserve isospin. Therefore f_1 decays are dominated by four-pion final states, which makes them harder to employ as an intuition building tool.

[2] A meson is labelled [1] "exotic" if it is characterised by a value of J^{PC} that is unobtainable in the $q\bar{q}$ constituent quark model; e.g., the $1.6\,\mathrm{GeV}$ $J^{PC}=1^{-+}$ state for which experimental evidence [4] has recently appeared. Such unusual charge parity states are a necessary feature of a field theoretical description of quark-antiquark bound states [2] with Bethe-Salpeter equation studies typically yielding [5] masses approximately twice as large as that of the natural charge parity partner and, in particular, a $J^{PC}=1^{-+}$ meson with a mass [6] of $\sim 1.4-1.5\,\mathrm{GeV}$.

integral, with Λ the regularisation mass-scale. In Eq. (1), S is the renormalised dressed-quark propagator and K is the renormalised, fully-amputated dressed-quark-antiquark scattering kernel. The equation has a solution Γ, the Bethe-Salpeter amplitude, only for particular, isolated values of P^2, which determine the mass of the associated bound state. The amplitude is a necessary element in the calculation of any of the meson's interactions.

The renormalised dressed-quark propagator in Eq. (1) is determined by the quark DSE[3]

$$S(p)^{-1} := i\gamma \cdot p\, A(p^2) + B(p^2) \tag{2}$$

$$= Z_2(i\gamma \cdot p + m^{\rm bm}) + Z_1 \int_q^\Lambda \tfrac{4}{3}\, g^2 D_{\mu\nu}(p-q)\gamma_\mu S(q)\Gamma_\nu(q,p), \tag{3}$$

where $D_{\mu\nu}(k)$ is the renormalised dressed-gluon propagator, $\Gamma_\nu(q,p)$ is the renormalised dressed-quark-gluon vertex, and $m^{\rm bm}$ is the Λ-dependent u-, d-current-quark bare mass. The renormalisation constants: $Z_1(\zeta^2,\Lambda^2)$ and $Z_2(\zeta^2,\Lambda^2)$, depend on the renormalisation point, ζ, and the regularisation mass-scale.

To proceed one must identify a reliable truncation of the integral kernels in Eqs. (1) and (3). Qualitatively reliable results are unobtainable unless the truncations ensure the preservation of the Ward-Takahashi identities that relate the dressed-quark propagator to the solution of the relevant inhomogeneous Bethe-Salpeter equations. Satisfying this constraint, results as important as the correlation between dynamical chiral symmetry breaking and the low mass of the pion (Goldstone's theorem) are automatic [9]. For most light mesons[4] the class of renormalisation-group-improved rainbow-ladder truncations is satisfactory. It is reliable in Landau gauge, a fixed point of the QCD renormalisation group, and defined by

$$Z_1\, g^2 D_{\mu\nu}(k-q)\,\Gamma_\nu(q,p) = \mathcal{G}(k-q)\, D_{\mu\nu}^{\rm free}(k-q)\,\gamma_\nu, \tag{4}$$

$$K_{tu}^{rs}(q,k;P) = -\mathcal{G}(k-q) D_{\mu\nu}^{\rm free}(k-q)\, \left(\tfrac{1}{2}\lambda^a\gamma_\mu\right)_{tr} \left(\tfrac{1}{2}\lambda^a\gamma_\nu\right)_{su}, \tag{5}$$

with $D_{\mu\nu}^{\rm free}(k) = (\delta_{\mu\nu} - k_\mu k_\nu/k^2)/k^2$ and $\mathcal{G}(k^2)/(4\pi) = \alpha(k^2)$ for $k^2 \gtrsim 1\,{\rm GeV}^2$. This truncation ensures that the solutions of the DSEs exhibit that momentum evolution characteristic of the QCD renormalisation group at one-loop order. For $k^2 \lesssim 1\,{\rm GeV}^2$, the form of $\mathcal{G}(k^2)$ is only loosely constrained and in contemporary phenomenological studies it is modelled, often in accordance with the constraints [11] supplied by studies of the gluon DSE [8,12].

An early, extensive study of the light meson spectrum using Eq. (1) in rainbow-ladder truncation is reported in Ref. [13]. It neglected axial-vector mesons. Improved studies, accounting for the complete Dirac structure of pseudoscalar [14] and vector [15] meson Bethe-Salpeter amplitudes and explaining many of the important contributions made by the nonleading components, such as the pseudovector component of the pion, have been

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[3]We use a Euclidean formulation with $\{\gamma_\mu,\gamma_\nu\} = 2\delta_{\mu\nu}$, $\gamma_\mu^\dagger = \gamma_\mu$, $p\cdot q = \sum_{i=1}^4 p_i q_i$, and ${\rm tr}_D[\gamma_5\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma] = -4\,\epsilon_{\mu\nu\rho\sigma}$, $\epsilon_{1234}=1$. A vector, k_μ, is timelike if $k^2<0$.

[4]The exceptions are the η-η' complex and light mesons that have vacuum quantum numbers; i.e., $J^{PC} = 0^{++}$, for which higher orders must be retained in the expansion of Γ_ν and K introduced and explored in Refs. [10].

completed, but the axial-vector mesons still await such a careful treatment. In its absence we use a crude, confining, Goldstone theorem preserving rank-2 separable Ansatz [5] for $\mathcal{G}(k^2)$ to provide guidance to the structure of the light mesons.

The foundation of the separable Ansatz is a model [16] dressed-quark propagator:

$$S(p) = -i\gamma\cdot p\,\sigma_V(p^2) + \sigma_S(p^2), \tag{6}$$

$$\bar\sigma_S(x) = 2\bar m_f \mathcal{F}(2(x+\bar m_f^2)) + \mathcal{F}(b_1 x)\mathcal{F}(b_3 x)\left[b_0 + b_2\mathcal{F}(\epsilon x)\right], \tag{7}$$

$$\bar\sigma_V(x) = \frac{1}{x+\bar m_f^2}\left[1-\mathcal{F}(2(x+\bar m_f^2))\right] = \frac{2(x+\bar m_f^2) - 1 + \mathrm{e}^{-2(x+\bar m_f^2)}}{2(x+\bar m_f^2)^2} \tag{8}$$

with: $\bar\sigma_S(x) = \lambda\,\sigma_S(p^2)$, $\bar\sigma_V(x) = \lambda^2\,\sigma_V(p^2)$; $\mathcal{F}(y) = (1-\mathrm{e}^{-y})/y$; $x = p^2/\lambda^2$; $\bar m_f = m_f/\lambda$; and λ a mass scale. This confining model efficiently characterises many essential and robust elements of the solution of Eq. (3), and has been used efficaciously [7] in a wide range of phenomenological applications: most recently in a survey of heavy meson observables [17] and an elucidation [18] of the effect of meson loops on ρ-meson properties.

In basing the separable Ansatz on this algebraic model, renormalisability is lost and a regularisation must be introduced to properly define the Dyson-Schwinger and Bethe-Salpeter equations. It is sufficient [5] to modify σ_S and σ_V so that $\mathcal{F}(b_1 x) \to \mathcal{F}((\epsilon_s x)^2)\mathcal{F}(b_1 x)$ in Eq. (7) and $[2(x+\bar m^2)-1] \to [2(x+\bar m^2) - \exp(-\epsilon_V^2(x+\bar m^2))]$ in Eq. (8). The parameters take the values

$$\begin{array}{ccccccc} \bar m & b_0 & b_1 & b_2 & b_3 & \epsilon_S & \epsilon_V \\ 0.00811 & 0.131 & 2.90 & 0.603 & 0.185 & 0.482 & 0.1 \end{array} \tag{9}$$

with $\lambda = 0.566\,\mathrm{GeV}$, and because it is a separable Ansatz the meson Bethe-Salpeter amplitudes are expressed solely in terms of two functions:[5]

$$\bar F(x) := \tfrac{1}{a}\left[\bar A(x) - 1\right],\ \bar G(x) := \tfrac{1}{b}\left[\bar B(x) - \bar m\right], \tag{10}$$

with calculated values of $a = 0.129$, $b = 0.0877$.

We can now solve the Bethe-Salpeter equation to obtain the masses and amplitudes of the participating mesons: the light axial-vectors, and the π, ρ and ω. Using the separable Ansatz the general form of their Bethe-Salpeter amplitudes is

$$\Gamma^\pi(\bar k;\hat P) = \vec\tau\,\gamma_5\left(ie_1^\pi - e_2^\pi\gamma\cdot\hat P\right)\bar G(x), \tag{11}$$

$$\Gamma^\rho_\mu(\bar k;\hat P) = \vec\tau\,\left(\bar k_\mu^T e_1^\rho\,\bar F(x) + ie_2^\rho\gamma_\mu^T\,\bar G(x) - ie_3^\rho\gamma_5\epsilon_{\lambda\mu\nu\sigma}\gamma_\lambda\bar k_\nu\hat P_\sigma\,\bar F(x)\right), \tag{12}$$

$$\Gamma^{b_1}_\mu(\bar k;\hat P) = \vec\tau\,\bar k_\mu^T\,\gamma_5\left(ie_1^{b_1} - e_2^{b_1}\gamma\cdot\hat P\right)\bar F(x), \tag{13}$$

$$\Gamma^{a_1}_\mu(\bar k;\hat P) = \vec\tau\,\left(i\gamma_5\gamma_\mu^T e_1^{a_1}\bar G(x) + ie_2^{a_1}\,\epsilon_{\lambda\mu\nu\sigma}\gamma_\lambda\bar k_\nu\hat P_\sigma\bar F(x)\right), \tag{14}$$

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[5]Detailed studies [13–15] show that this is flawed; i.e., in general, the momentum dependence of the modulating functions in the Bethe-Salpeter amplitudes is **not** related to the scalar functions in the quark propagator in such a trivial fashion. However, we only require that the separable Ansatz provide initial guidance and herein also report results obtained with a rudimentary amelioration of this defect.

where $\bar{k} = k/\lambda$, $\hat{P}_\mu = P_\mu/|P^2|^{1/2}$, $\bar{k}^T_\mu = (\bar{k}_\mu + \hat{P}_\mu \bar{k}\cdot\hat{P})$, and $\gamma^T_\mu = (\gamma_\mu + \hat{P}_\mu \gamma\cdot\hat{P})$. Here the eigenvectors $\vec{e}^{\,H}$, obtained by solving the Bethe-Salpeter equation, are all that remain in the separable Ansatz of the detailed structure exhibited by a realistic Bethe-Salpeter amplitude. Qualitative information nevertheless remains; e.g., comparing the π and b_1 amplitudes it is apparent that the b_1 has characteristics consistent with an orbital excitation. Further, even using this Ansatz one observes that the momentum dependence of the scalar functions modulating the amplitudes is very different: $\bar{G}$ for π, $\bar{F}$ for b_1, a sharp contrast with constituent-quark-model-like treatments. The Bethe-Salpeter amplitudes of the ω, h_1, f_1 are obtained from those of the ρ, b_1, a_1 via the replacement $\vec{\tau} \to \tau^0 := \mathrm{diag}(1,1)$. These partners are degenerate in rainbow-ladder truncation (cf. experimentally, none of the mass differences is greater than 5% of the average mass).

The separable Bethe-Salpeter equation is easily solved and yields the following masses and eigenvectors (quoted here as normalised trivially via $\Sigma e_i^2 = 1$)

	$m_{\rm exp}$(GeV)	m(GeV)	e_1	e_2	e_3
π	0.139	0.139	0.9973	-0.0732	
ω, ρ	0.77	0.736	-0.2270	0.9610	-0.1580
h_1, b_1	1.23 ± 0.003	1.24	0.9708	0.2400	
f_1, a_1	1.23 ± 0.040	1.34	0.1991	0.9800	

(15)

thus reproducing some of the results in Ref. [5] and providing the first calculation of the b_1. (The observed [1] isovector masses are quoted for comparison.) Here, as with variational wave functions obtained in the estimation of energy levels, an accurate value of the mass does not necessarily mean that the model Bethe-Salpeter amplitude will be reliable for the calculation of transitions. Nevertheless, the suggestions about the form cannot be ignored, and it is interesting and important to observe that the $\gamma_5\gamma_\mu$ term in the a_1 amplitude is **not** the dominant contribution, which is provided instead by the second term that exhibits characteristics of orbital motion.

The correct, canonical normalisation of the Bethe-Salpeter amplitude ensures that the pole associated with the meson in the quark-antiquark scattering amplitude: $M := K + K(SS)K + \ldots$, has unit residue. In calculations of observables one must therefore rescale the naively normalised eigenvectors in Eq. (15) by a constant factor: $e_i^H \to e_i^H/N_H$, which for the pion is obtained in rainbow-ladder truncation from

$$2N_\pi^2\,\delta^{ij}\,P_\mu = \tag{16}$$
$$\mathrm{tr}\int_q^\Lambda \left[\Gamma^{\pi i}(\bar{q};-\hat{P})\frac{\partial S(q_+)}{\partial P_\mu}\Gamma^{\pi j}(\bar{q};\hat{P})S(q_-) + \Gamma^{\pi i}(\bar{q};-\hat{P})S(q_+)\Gamma^{\pi j}(\bar{q};\hat{P})\frac{\partial S(q_-)}{\partial P_\mu}\right]\Bigg|_{P^2=-m_\pi^2},$$

where $i,j = 1,2,3$ label the Pauli matrices in Eq. (11) and the trace is over colour, Dirac and isospin indices, and for the $J=1$ mesons from ($\alpha,\beta = 0,1,2,3$)

$$2N_H^2\,\delta^{\alpha\beta}\,P_\mu = \tag{17}$$
$$\tfrac{1}{3}\mathrm{tr}\int_q^\Lambda \left[\Gamma^{H\alpha}_\nu(\bar{q};-\hat{P})\frac{\partial S(q_+)}{\partial P_\mu}\Gamma^{H\beta}_\nu(\bar{q};\hat{P})S(q_-) + \Gamma^{H\alpha}_\nu(\bar{q};-\hat{P})S(q_+)\Gamma^{H\beta}_\nu(\bar{q};\hat{P})\frac{\partial S(q_-)}{\partial P_\mu}\right]\Bigg|_{P^2=-m_H^2}.$$

Our primary focus is the pionic decays of the axial-vector mesons. The decay $a_1(P_\mu) \to \rho(q_\nu)\pi(k)$, $P^2 = -m_{a_1}^2$, $q^2 = -m_\rho^2$, $k^2 = -m_\pi^2$, is described by an amplitude

$$T^{a_1\rho\pi}_{\mu\nu} = \mathcal{A}(m_{a_1}^2, m_\rho^2)\, G_{\mu\nu} + \mathcal{B}(m_{a_1}^2, m_\rho^2)\, L_{\mu\nu}, \tag{18}$$

that we have expressed in terms of two projection operators ($Y = (P\cdot q)^2 - m_{a_1}^2 m_\rho^2$):

$$G_{\mu\nu} = \delta_{\mu\nu} - \frac{1}{Y}\left[m_{a_1}^2 q_\mu q_\nu + m_\rho^2 P_\mu P_\nu + P\cdot q\,(P_\mu q_\nu + q_\mu P_\nu)\right], \tag{19}$$

$$L_{\mu\nu} = \frac{P\cdot q}{Y}\left(P_\mu + q_\mu \frac{m_{a_1}^2}{P\cdot q}\right)\left(q_\nu + P_\nu \frac{m_\rho^2}{P\cdot q}\right), \tag{20}$$

with the properties: $P_\mu G_{\mu\nu} = 0 = G_{\mu\nu} q_\nu$, $P_\mu L_{\mu\nu} = 0 = L_{\mu\nu} q_\nu$, a manifestation of the on-shell transversality of $J=1$ mesons; $G_{\mu\nu}G_{\mu\nu} = 2$; $L_{\mu\nu}L_{\mu\nu} = m_{a_1}^2 m_\rho^2/(P\cdot q)^2$; and $G_{\mu\nu}L_{\mu\nu} = 0$. The scalar functions: $\mathcal{A}(m_{a_1}^2, m_\rho^2)$ and $\mathcal{B}(m_{a_1}^2, m_\rho^2)$, contain all the dynamical information about this process; e.g., from Eq. (18) one obtains

$$\Gamma_{a_1\rho\pi} = \frac{1}{12\pi}\frac{|\vec{k}|}{m_{a_1}^2}\left(2|\mathcal{A}|^2 + \frac{m_{a_1}^2 m_\rho^2}{(P\cdot q)^2}|\mathcal{B}|^2\right), \tag{21}$$

with $|\vec{k}|^2 = \lambda(m_{a_1}^2, m_\rho^2, m_\pi^2)/(2m_{a_1})^2$, $\lambda(m_{a_1}^2, m_\rho^2, m_\pi^2) = (m_{a_1}^2 - (m_\rho + m_\pi)^2)(m_{a_1}^2 - (m_\rho - m_\pi)^2)$, and following Ref. [19] the ratio

$$D/S|_{a_1\rho\pi} = \frac{f^D_{a_1\rho\pi}}{f^S_{a_1\rho\pi}} \tag{22}$$

where

$$f^S_{a_1\rho\pi}(m_{a_1}^2, m_\rho^2) = \frac{\sqrt{4\pi}}{3m_\rho}\left[(E_\rho + 2m_\rho) f_{a_1\rho\pi}(m_{a_1}^2, m_\rho^2) + |\vec{k}|^2 m_{a_1} g_{a_1\rho\pi}(m_{a_1}^2, m_\rho^2)\right], \tag{23}$$

$$f^D_{a_1\rho\pi}(m_{a_1}^2, m_\rho^2) = -\frac{\sqrt{8\pi}}{3m_\rho}\left[(E_\rho - m_\rho) f_{a_1\rho\pi}(m_{a_1}^2, m_\rho^2) + |\vec{k}|^2 m_{a_1} g_{a_1\rho\pi}(m_{a_1}^2, m_\rho^2)\right], \tag{24}$$

with $E_\rho^2 = |\vec{k}|^2 + m_\rho^2$, $f_{a_1\rho\pi}(m_{a_1}^2, m_\rho^2) = \mathcal{A}(m_{a_1}^2, m_\rho^2)$; and

$$g_{a_1\rho\pi}(m_{a_1}^2, m_\rho^2) = \frac{P\cdot q}{Y}\left(-\mathcal{A}(m_{a_1}^2, m_\rho^2) + \mathcal{B}(m_{a_1}^2, m_\rho^2)\frac{m_{a_1}^2 m_\rho^2}{(P\cdot q)^2}\right). \tag{25}$$

We calculate $T^{a_1\rho\pi}_{\mu\nu}$ in impulse approximation: [6]

$$iT^{a_1\rho\pi}_{\mu\nu}(P,q) = 2\,\mathrm{tr}\int_\ell^\Lambda S(\ell_{--})\Gamma^{a_1}_\mu(\bar{\ell};-\hat{P})S(\ell_{++})\Gamma^\pi(\bar{\ell}_{+0};\hat{k})S(\ell_{+-})\Gamma^\rho_\nu(\bar{\ell}_{0-};\hat{q}) \tag{26}$$

where $\ell_{\alpha\beta} = \ell + \alpha q/2 + \beta k/2$. It is the simplest approximation consistent with the rainbow-ladder truncation and corrections can be incorporated systematically [10]. Calculations [20] indicate that they contribute $\lesssim 15\%$ for on-shell momenta. With our estimate of the

[6] The analogous approximation to the matrix element for the $f_1 \to$ vector-meson-plus-pion decay is easily shown to yield $\mathcal{A} \equiv 0 \equiv \mathcal{B}$.

meson Bethe-Salpeter amplitudes and form for the dressed-quark propagator, Eq. (26) is straightforward to evaluate, remembering that here and below the Bethe-Salpeter amplitudes are rescaled by N_H, as discussed in connection with Eqs. (16) and (17).

The $b_1(P) \to \omega(q)\pi(k)$ matrix element also has the representation in Eq. (18) and in impulse approximation is

$$T^{b_1\omega\pi}_{\mu\nu}(P,q) = 2\,\mathrm{tr}\int_\ell^\Lambda S(\ell_{--}) i\Gamma^{b_1}_\mu(\bar\ell;-\hat P)S(\ell_{++})i\Gamma^\pi(\bar\ell_{+0};\hat k)S(\ell_{+-})\Gamma^\omega_\nu(\bar\ell_{0-};\hat q), \tag{27}$$

with the width given by

$$\Gamma_{b_1\omega\pi} = \frac{1}{24\pi}\frac{|\vec k|}{m^2_{b_1}}\left(2|\mathcal{A}(m^2_{b_1},m^2_\omega)|^2 + \frac{m^2_{b_1}m^2_\omega}{(P\cdot q)^2}|\mathcal{B}(m^2_{b_1},m^2_\omega)|^2\right), \tag{28}$$

$|\vec k|^2 = \lambda(m^2_{b_1},m^2_\omega,m^2_\pi)/(2m_{b_1})^2$, and $D/S|_{b_1\omega\pi}$ obtained by analogy with Eqs. (22)-(24).

$h_1(P)\to\rho(q)\pi(k)$ is directly accessible too, with the impulse approximation to its matrix element obtained via the obvious modifications of Eq. (27): $b_1\to h_1$, $\omega\to\rho$, the width given by

$$\Gamma_{h_1\rho\pi} = \frac{1}{8\pi}\frac{|\vec k|}{m^2_{h_1}}\left(2|\mathcal{A}(m^2_{h_1},m^2_\rho)|^2 + \frac{m^2_{h_1}m^2_\rho}{(P\cdot q)^2}|\mathcal{B}(m^2_{h_1},m^2_\rho)|^2\right), \tag{29}$$

$|\vec k|^2 = \lambda(m^2_{h_1},m^2_\rho,m^2_\pi)/(2m_{h_1})^2$, and $D/S|_{h_1\rho\pi}$ obtained as the obvious analogue of Eq. (22).

Our results for these strong decays can be placed in perspective by comparing them with those for the $\rho(P)\to\pi(q)\pi(k)$ decay, whose matrix element can be written

$$M_\mu(k,q) = (k-q)_\mu f^+(t) + P_\mu f^-(t), \tag{30}$$

$t=-(k-q)^2$, in which case

$$\Gamma_{\rho\pi\pi} = \frac{g^2_{\rho\pi\pi}}{6\pi}\frac{|\vec k|^3}{m^2_\rho},\; g_{\rho\pi\pi} = \tfrac{1}{2}f^+(t=m^2_\rho), \tag{31}$$

$f^-(m^2_\rho)\equiv 0$, $|\vec k|^2 = \lambda(m^2_\rho,m^2_\pi,m^2_\pi)/(2m_\rho)^2$. The impulse approximation to $M_\mu(k,q)$ is

$$iM_\mu(k,q) = \mathrm{tr}\int_\ell^\Lambda S(\ell_{--})\Gamma^\rho_\mu(\bar\ell;-\hat P)S(\ell_{++})\Gamma^\pi(\bar\ell_{+0};\hat k)S(\ell_{+-})\Gamma^\pi(\bar\ell_{0-};\hat q). \tag{32}$$

This perspective is sharpened by also calculating the meson decay constants, which for $J=1$ are given in QCD by [17]

$$i\sqrt{2}\,\delta^{\alpha\beta}\,f_H\,m_H = \tfrac{1}{3}\mathrm{tr}Z_2\int_k^\Lambda(\gamma_\mu-\gamma_\mu\gamma_5)\tau^\alpha S(k_+)\Gamma^{H\beta}_\mu(\bar k;\hat P)S(k_-). \tag{33}$$

These decay constants are the analogue of those associated with pseudoscalar mesons, the expression for which is given in Ref. [14], and completely describe [15,17] the strong interaction contribution to the weak decays of the charged mesons and electromagnetic decays of the neutral.[7] We note that because of the charge-parity of the h_1, b_1 mesons, which is manifest in the Dirac component of their Bethe-Salpeter amplitudes via:

$$\bar\Gamma^H_\mu(\bar k;\hat P) := \left(C^{-1}\Gamma^H_\mu(-\bar k;\hat P)C\right)^T = -\Gamma^H_\mu(\bar k;\hat P), \tag{34}$$

$C = \gamma_2\gamma_4$ is the charge-conjugation matrix, it follows as a model-independent result from Eq. (33) that

$$f_{h_1} \equiv 0 \equiv f_{b_1}. \tag{35}$$

This simply reflects the fact that a $V-A$ operator cannot connect a 1^{+-} state to the 0^{++} vacuum.

For the first of our calculations all the necessary elements are now defined: the model dressed-quark propagator, Eqs. (6)-(9); and the Bethe-Salpeter amplitudes, Eqs. (10)-(17). With these amplitudes, which alone are regularised as described before Eq. (9), we have a simple model in which the meson decay constants are finite and $Z_2 \to 1$. A direct calculation yields the results in column I of Table I. Superficially: the decay constants are broadly acceptable, the D/S ratios have the correct sign, and the widths are too large, resulting from an overestimate of the couplings $\mathcal{A}$, $\mathcal{B}$, $g_{\rho\pi\pi}$ by a factor of ~ 1.5–3.0. Delving further we find that the results are very sensitive to the eigenvectors in Eq. (15) and the form of the scalar functions that modulate the different components of the Bethe-Salpeter amplitude. These are model-dependent features that are determined by the infrared behaviour of the interaction; i.e., the form of $\mathcal{G}(k^2)$ for $k^2 \lesssim 1\,\mathrm{GeV}^2$. In a sense this is in qualitative agreement with the observation [3] quoted in the introduction; i.e., that the D/S ratios are sensitive to the long-range form of the quark-antiquark interaction.

It is important to observe that the values of the D/S ratios and the ratio of these ratios demonstrate that the class of rainbow-ladder DSE models should **not** be confused with the class of one-gluon exchange models used extensively in constituent-quark-like models of hadrons, which necessarily yield [3] the **wrong** sign for the D/S ratios. It is incorrect to anticipate defects in the former based on those of the latter. We are able to obtain the wrong sign for the D/S ratios; e.g., forcing $e_1^{a_1} = 0$ with no other modification yields: $D/S|_{a_1\rho\pi} = 0.27$, $\Gamma_{a_1\rho\pi} = 0.141\,\mathrm{GeV}$ and $f_{a_1} = 0.154\,\mathrm{GeV}$. However, since the Bethe-Salpeter amplitudes are dynamically determined this simply confirms that the ratios do indeed provide useful constraints. This sensitivity to details of the Bethe-Salpeter amplitude is another example of that observed in Ref. [5] and emphasised by the results described in Refs. [21].

An obvious question arises: is there a model for the scalar function $\mathcal{G}(k^2)$ that can provide a good description of the observables; i.e., are these phenomena describable using the rainbow-ladder truncation of the DSEs with no additional mechanism? The success of studies such as Refs. [14,15] suggest that they are. Here we follow a standard approach

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[7]The factor $Z_2(\zeta^2,\Lambda^2)$ in Eq. (33) ensures that the right-hand-side is finite as $\Lambda \to \infty$ and is renormalisation-point and gauge-parameter independent. The manner in which this is realised in QCD is nevertheless interesting, requiring a conspiracy between various components of the meson Bethe-Salpeter amplitudes, and is elucidated in Ref. [15].

and address a simpler question. Since $\mathcal{G}(k^2)$ determines the dressed-quark propagator and the Bethe-Salpeter amplitudes, then modelling these elements instead provides a *de facto* model of $\mathcal{G}(k^2)$: after all, that is the rationale behind separable Ansätze [5,22]. To relax the constraints imposed by the rank-2 separable Ansatz we allow $b_2^+ := b_2^{a_1} = b_2^{b_1}$, b_2^π, b_0^ρ, b_1^ρ and $e_2^{b_1}$ to vary in the calculation of the Bethe-Salpeter amplitudes and optimise a least-squares fit to the underlined quantities in the table. This simple expedient admits some of those differences between the modulating functions observed in all of the more sophisticated BSE studies and the possibility of remediating the separable kernel's simplistic Chebyshev expansion.

An optimal fit is obtained with

$$\begin{array}{ccccc} b_2^+ & b_2^\pi & b_0^\rho & b_1^\rho & e_2^{b_1} \\ 0.863 & 0.891 & 0.690 & 3.43 & -0.673, \end{array} \tag{36}$$

$(e_1^{b_1})^2 + (e_2^{b_1})^2 = 1$, and yields the results presented in column II, Table I, which has a rms relative error of 17%. (cf. 30% over five comparable items in Table I of Ref. [3].) The most significant effect of the relaxation is the change in sign and relative magnitude of $e_2^{b_1}$. Such a quantitative change is not too surprising given that Γ^{b_1} is the Bethe-Salpeter amplitude most affected by shortcomings in the angular projection of the rank-2 separable Ansatz. Thus a satisfactory understanding of these phenomena is possible in our approach. However, it is clear that a definitive study combining and extending Refs. [14,15] is required to conclusively answer the question posed above.

We have applied a simple DSE model to the calculation of the spectrum and decays of light axial-vector mesons. Like light and heavy vector meson observables [17], they find a natural explanation in the momentum-dependent dressing of quark propagators and the detailed form of the meson Bethe-Salpeter amplitudes. In the more sophisticated versions of this approach these qualitatively important features are tied to the long-range form of the interquark interaction, which in Refs. [13–15] is represented via the infrared behaviour of $\mathcal{G}(k^2)$: the scalar function that characterises the renormalisation-group-improved rainbow-ladder truncation. (The ultraviolet behaviour of S, Γ_ν and Γ^H is model independent.)

There are phenomena that cannot be described using the rainbow-ladder truncation: $J^{PC} = 0^{++}$ meson masses and the η-η' complex being significant examples, and another the critical exponents [23] of the finite temperature chiral symmetry restoring transition. However, a qualitative understanding of the reasons underlying these failures exists so that the truncation's domain of applicability is becoming properly demarcated. That allows for the sensible interpretation of experimental results in terms of $\mathcal{G}(k^2)$, and thereby provides a tool for the critical evaluation of contemporary numerical estimates [24] and, where required, improvements in the truncation of K, the quark-antiquark scattering kernel.

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TABLES

TABLE I. Calculated results compared with observed or inferred values, which are taken from Refs. [1,17,19]. We used the calculated masses in Eq. (15) with the exception of $m_{h_1} = 1.17\,\mathrm{GeV}$ and all dimensioned quantities are quoted in GeV. Column I: calculated using exactly those Bethe-Salpeter amplitudes obtained with the separable Ansatz for $\mathcal{G}(k^2)$ in Eqs. (4), (5); Column II: optimised least-squares fit to the underlined quantities obtained using a simple expedient that promotes further diversity between the meson Bethe-Salpeter amplitudes.

	Obs.	I	II
$D/S\|_{h_1\to\rho\pi} =: R_{h_1}$		0.81	0.25
$D/S\|_{a_1\to\rho\pi} =: \underline{R_{a_1}}$	-0.1 ± 0.028	-0.092	-0.075
$D/S\|_{b_1\to\omega\pi} =: \underline{R_{b_1}}$	0.29 ± 0.04	0.97	0.31
R_{a_1}/R_{b_1}	-0.34 ± 0.11	-0.095	-0.25
R_{h_1}/R_{b_1}		0.84	0.83
$\underline{g_{\rho\pi\pi}}$	6.05 ± 0.02	9.58	8.18
$\Gamma_{\rho\to\pi\pi}$	0.151 ± 0.001	0.356	0.259
$\underline{\Gamma_{a_1\to\rho\pi}}$	0.25 - 0.60	4.02	0.385
$\underline{\Gamma_{b_1\to\omega\pi}}$	0.142 ± 0.009	0.308	0.146
$\Gamma_{h_1\to\rho\pi}$	0.360 ± 0.040	0.573	0.301
$\underline{f_{a_1}}$	0.203 ± 0.018	0.121	0.221
$\underline{f_\rho}$	0.216 ± 0.005	0.189	0.223
$\underline{f_\pi}$	0.1307 ± 0.0004	0.136	0.148